Modeling Frost Line Soil Penetration Using Freezing Degree-Day Rates, Day Length, and Sun Angle

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ABSTRACT

Seasonal soil freezing depth varies from year to year, and knowledge of frost line depth and progression is critical to a number of applications. Frost depth was measured using a CRREL-Gandahi frost tube over a period of three years (2008–2011). The frost tubes provided a measure of frost depth to verify model outputs and were installed at two locations: Buffalo, NY (Buffalo State College), and at a Cooperative Observing Station (COOP) site in North Tonawanda, NY. With the exception of one set of frost tubes, the area around each frost tube was routinely cleared of snow. Freezing degree-days were calculated from nearby weather stations. A model was developed to forecast the progression and depth of the frost line based on freezing degree-day rates (FDD/day), with adjustments for day length and sun angle based on frost measurement in 2008–2010 and validated for 2010–2011. The frost line climate-only model proved itself a simple to use model that required easily obtained inputs and provides excellent agreement with measured values. The model is best used to predict frost depths in areas and times where snow cover is absent or when potential frost depth maximums are to be determined.

Keywords: modeling, soil frost depth, frost line, climate, freezing degree-days, day length, sun angle, Buffalo New York.

INTRODUCTION

Seasonally frozen ground undergoes a freeze-thaw cycle every year. Much of the contiguous United States, with the exception of the most southern states and western California, experiences some level of seasonal soil freezing (Davis, 2001). Farther north and at high elevations, the freezing depth, referred to as the frost line, reaches greater depths and eventually transitions to permanently frozen ground (permafrost) in Northern Canada and Alaska.

Information on real-time seasonal soil freezing depth is critical to a number of applications, including construction codes that dictate the depth of building foundations (Burn, 1976; ICC, 2011); the laying of pipe (Sepehr and Goodrich, 1994; Rajani and Kleiner, 2001); the construction of landfill covers (Smith and Rager, 2002); the determination of plant hardiness, root injury, and weed survival (Kahimba et al., 2009; Boydston et al., 2006); precipitation and snowmelt runoff (Bayard et al., 2004; Vasilyev, 1994); and even the frost heaving of buried artifacts (Johnson and Hansen, 1974). While a general approximation of frost depth can be made, often based on extremes, the progression of the frost line and its maximum depth in seasonally frozen ground can vary considerably from year to year.

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There exist a number of direct approaches to determine frost depth, including the digging of a soil pit to observe the presence of ice crystals, the burying of moisture blocks, the recording of soil conductivity, and the burying of thermocouples, to name a few. Another approach is the installation of frost tubes (McCool and Molnau, 1984; Hardy, et al., 2001). Each of these approaches requires in-situ observations through the soil column. The simplest approach is the use of the Air Freezing Index (AFI), where freezing degree-days are accumulated and used as a surrogate for frost depth (Burn, 1976; Hardy et al., 2001; and Berry, 1998). Other approaches infer frost depth based on the modeling of physical processes related to the freezing of soil, such as the Simultaneous Heat and Water (SHAW) model (Kahimba et al., 2009; and Kennedy and Brenton, 1998), or soil-based heat flux balance approaches (Benoit and Mostaghimi, 1985), or hybrids (DeGaetano et al., 2001).

Of the numerous approaches available in the literature, the AFI remains the simplest to use—AFI versus freezing depth. While the AFI provides a rough estimate of frost depth, it is not the best indicator as it tends to over-predict frost penetration. Vermette and Christopher (2008) have shown that it is not the AFI but the AFI rate (referred to as freezing degree-days per day or FDD/day) that provides a good surrogate to predict the timing and depth of the frost line. The objective of this study is to develop a simple to use, climate-only model that calculates the progression and depth of the frost line using a FDD/day rate approach.

METHODS

Measuring soil frost depth

CRREL-Gandahl frost tubes (Ricard et al., 1976) were used to measure soil freezing depth. These tubes have been shown to provide reasonably accurate frost depth at a low cost. A disadvantage is a reported time lag of 2 to 4 days and a maximum depth lag of 5 cm (McCool and Molnau, 1984). The frost tubes were constructed of 3.2-mm (1/8-in.) thick clear Tygon tubing, with an outer dimension of 15.9 mm (5/8 in.). A nylon string ran up the center of the tube to secure the frozen liquid. Rubber bands secured the string to rubber stoppers at either end of the tube. The tubes were constructed to a length of 1.52 m following a design first described by Ricard et al. (1976) and McCool and Molnau (1984) and used by Vermette and Christopher (2008). The tubes were filled with a solution of methylene blue (0.5 g/L). Freezing depth was indicated by a color change in the dye from blue (thawed) to clear (frozen). McCool and Molnau (1989) noted the ease with which the color change is located and read within the tube. An auger was used to drill a 91 cm deep hole into the soil into which a 3.2-cm (1.25-in.) wide PVC pipe was tightly placed. The lower end of the pipe was capped. A frost tube was inserted into the PVC pipe, allowing for about 51 cm of the frost tube to rise above the surface. The ground level was marked on the frost tube. The top of the PVC pipe extended 60 cm above ground level. The top of the frost tube was attached to an eye hook that was screwed into the upper cap of the PVC pipe. This attachment prevented the frost tube from sagging while positioned within the PVC pipe and allowed for easy raising and lowering of the frost tube when taking measurements. The length of the color change from the ground level downward was measured as the freezing depth (Figure 1).

Figure 1. Frost tube measurement showing a frost depth of 8.8 cm (3.5 in.). The lower edge of the tape indicates ground level.

The frost tubes were installed at two locations within western New York, United States. A group of four frost tubes was installed in a fenced area on the grounds of Buffalo State College (42° 56’ 9.5568” N and 78° 53’ 0.7908” W), located within the city of Buffalo, NY (Erie County), hereafter referred to as Site 1. At Site 1, two frost tubes were installed in an area that was cleared of snow daily to a radius of 2 m. Two additional frost tubes were installed in an adjacent area that was not cleared of snow. The Site 1 frost tubes were monitored during the winter of 2008–2009. The second location, hereafter referred to as Site 2, was a National Weather Service (NWS) Cooperative Observing (COOP) site in North Tonawanda, NY (Niagara County) (43^{o} 1’ 18.5406” N and 78^{o} 50’ 47.5” W). The two sites are located approximately 10 km apart (Figure 2). A single frost tube was installed at Site 2 and snow was removed from a 2-m radius daily, often two or three times a day during periods of extended snowfall (Figure 3). The Site 2 frost tube was monitored over three consecutive winters, from 2008 to 2011.

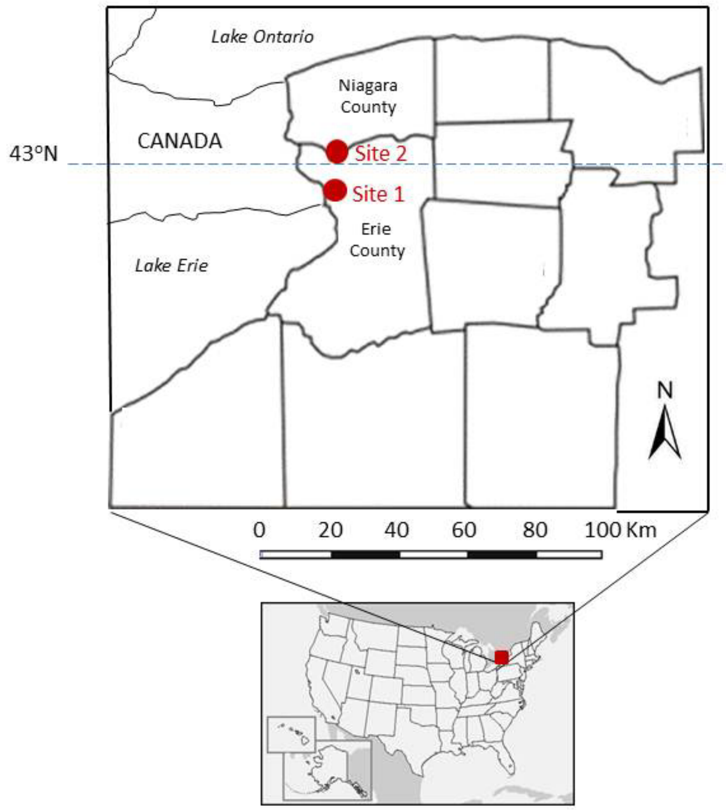

Figure 2. Site locations in Niagara and Erie Counties, Western New York, United States.

Figure 3. A frost tube (outer PVC casing showing) in a snow cleared area at Site 2.

Both sites typified a grassy surface, with an upper layer of soil/loam to a depth of about 25 cm and an under layer of clay (Figure 4). The frost tubes were initially installed in 2008, a few weeks before the onset of freezing air temperatures, and removed each spring for draining and storage. The Site 2 PVC pipe was capped and left in the ground between seasons.

Figure 4. Soil Profile at Site 1. The upper 25 cm are loam/soil with an under layer of clay (about 5 cm of clay are showing in the photograph). The soil pit was dug in the summer months.

Calculating degree-days

Ambient air temperatures were taken from onsite weather stations. Temperatures for Site 1 were taken from the Buffalo State College Vantage Pro weather station (Davis Instruments) located about 200 m from Site 1. Mean daily temperatures were calculated as the average of 24 hourly values. Site 2 temperature data were taken from a NWS COOP site located only a few meters from the frost tube. At Site 2 the mean daily temperatures were calculated as the average of the daily minimum and maximum thermometer readings (read at midnight). Both approaches are generally used to calculating mean daily temperatures.

Freezing degree-days were calculated by subtracting mean daily temperatures from 32°F. The Fahrenheit scale was used in this study as it is the temperature scale reported by the NWS and used by its operators. The Fahrenheit FDD has been converted to a Celsius FDD (dividing the former by 1.8) on all pertinent figures and tables in this paper. A negative value was recorded as a freezing degree-day (FDD), while a positive value was recorded as a thawing degree-day (TDD). A running total of degree-days were maintained through the winter season—November through March—where accumulated FDDs increase with added FDDs and decrease with added TDDs.

Modeling real-time frost penetration

The modeling approach of this study builds on the assumption that the dominant control on the progression and depth of the frost line is air temperature and builds on the earlier work of Vermette and Christopher (2008), where the FDD/day rate was shown a better measure of frost depth than accumulated FDDs (AFI). Based on two winters (2008–2010) of field data (FDDs and frost depth), the initial model consisted of a "rating curve" where measured FDD/day rates were paired with measured frost depths and plotted. The assumption was that the larger FDD/day rate has a greater potential to push the frost line deeper into the soil. Each days FDDs were multiplied by the plotted rate to calculate an "initial model" frost depth. The "final model" included adjustments for day length and sun angle, referred to as "solar adjustments." These adjustments accounted for surface soil warming that limited the freezing penetration. The model was validated using 2010–2011 measured frost depths.

RESULTS AND DISCUSSION

Frost depth measurements with no snow cover

The frost tubes were easy to read (a color change was distinct at the line of freezing) and appeared to work well. Surface soil conditions, as to whether frozen or thawed, were noted simply by the feel of the soil at the surface. A test pit was dug in proximity to Site 2. It confirmed a frost line at a similar depth (45 cm) to that recorded by the frost tube. A comparison of adjacent frost tubes at Site 1 showed no significant difference (Z-test p = 0.05) in mean values with absolute differences of no more than 1 to 2 cm for readings taken on the same day. The one exception occurred in late January 2009 when the difference between frost tubes grew to 3 cm over a period of one week. At Site 2, readings were taken at both midnight and noon. While the midnight data were used in this paper, no significant differences (Z-test p = 0.05) were found between the midnight and noon values. Given the lag depth, as reported by McCool and Molnau (1984), these absolute values are well within expected ranges.

Within the three year monitoring period from 2008 to 2011, the maximum depth of frost penetration was recorded at 45.0 cm. Each of the annual maximum depths occurred in late January and in the first two weeks of February (Table 1). In contrast, the maximum number of FDDs occurred in late February and the first week of March. As reported by Vermette and Christopher (2008), the FDDs over-predicted the penetration of the frost line.

Table 1. Summary data (freezing depths and FDDs) and dates. The maximum number of FDDs are given as both a Fahrenheit FDD and a Celsius FDD.

Site	Winter Season	Freezing Soil Depth in cm (in)	Date Max. Depth	Max. FDD Fah. Cel.	Date Max. FDD
Site 1	2008–2009	29.5 (11.6)	February 6	469 261	March 4
Site 2	2008–2009	40.5 (15.9)	January 29	652 362	March 4
Site 2	2009–2010	27.5 (10.8)	February 14	521 289	February 27
Site 2	2010–2011	45.0 (18.8)	February 14	709 394	March 3

The daily progression of FDDs and measured frost depths are shown in Figures 5 and 6. After a period of initial oscillation, frost depths are shown to progressively penetrate deeper into the soil from about the first week of January to mid-February. The FDDs followed a similar pattern. This period of deepening frost depth is referred to here as the “winter core,” where frost depths and accumulated FDDs are well correlated (R^2 = 0.99 [2008–2009, Site 1]; R^2 = 0.98 [2008–2009, Site 2]; R^2 = 0.93 [2009–2010]; R^2 =0.98 [2010–2011]). Exceptions do occur during mid-winter thaws, where the frost depth decreases along with surface thawing (see 2010–2011 in Figure 6). After this core period, frost depths gradually lessen over a period of about one month while the FDDs continue to accumulate. The point at which the soil thaws at depth does not occur when FDD’s are countered by an equal number of accumulated TDD’s but, rather, when TDD’s dominate over FDD’s such that the accumulated FDD value changes direction (starts returning to zero). This change in direction usually occurs in early March and lasts about 10 days before the soil is thawed at depth (see Figure 5).

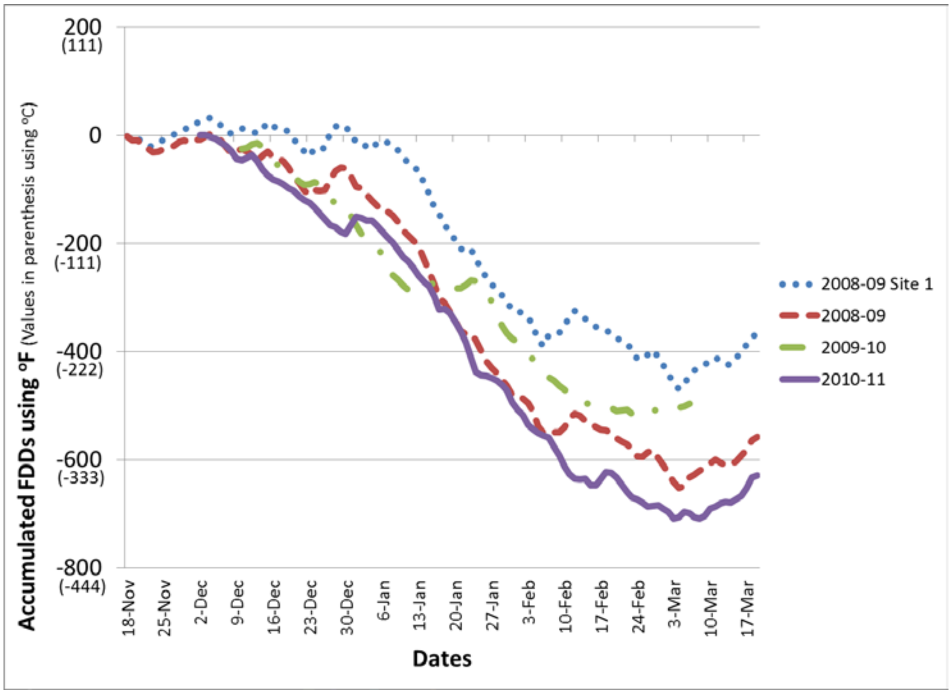

Figure 5. Accumulated freezing degree-days (FDDs), expressed as Fahrenheit FDDs and Celsius FDDs. Site 1 is labeled; remaining years are for Site 2.

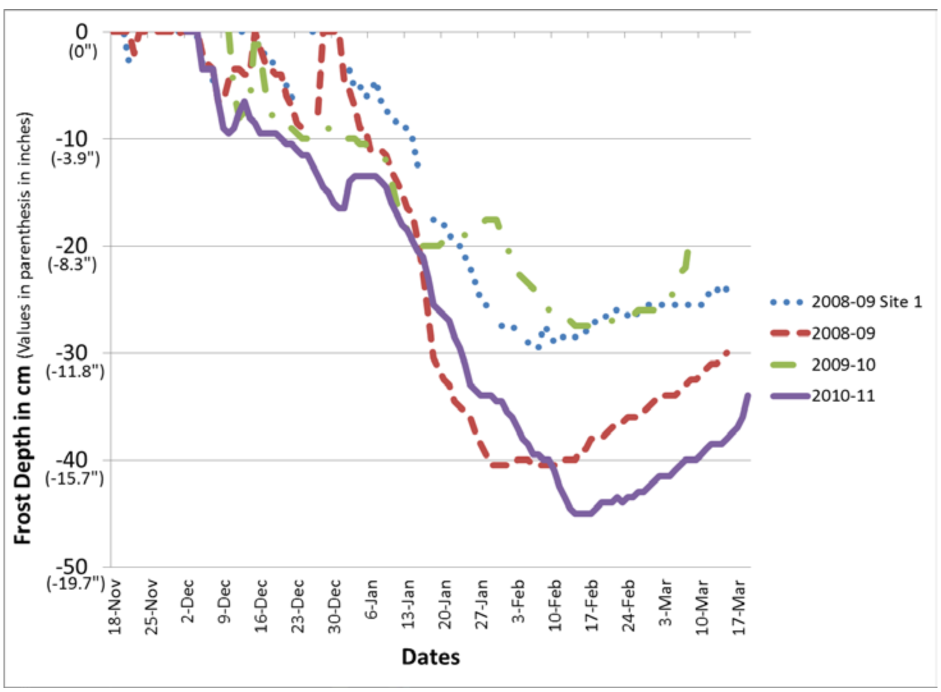

Figure 6. Progression of freezing depths. Site 1 is labeled (missing data shown as gaps for 2008–2009), remaining years are for Site 2.

The difference in frost penetration between the two sites in 2008–2009 can be attributed to their differing location. Site 1 experienced both a frost depth and accumulated FDDs that were less than Site 2. This difference is expected, as Site 1 is located closer to Lake Erie (moderating temperatures) and within an urban heat island. While absolute values differ between the two sites, the accumulated FDDs show a strong correlation (R^2 = 0.99). The penetration of the frost line, while differing in absolute values, is also comparable (R^2 = 0.97), and the date of the accumulated FDD maximum is the same for both sites (Table 1).

Frost depth measurement with snow cover

A total of 127.5 cm of snow fell at Site 1 in 2008–2009, with most of it falling from mid-December to late January. The month of February recorded only 3.75 cm of snow. The deepest snowpack was 36.25 cm, occurring in late January. By 10 February, the snow cover was nonexistent and remained so throughout the winter. While the snow-cleared Site 1 frost tube showed a frost line penetration from 29.5 to 32.0 cm through the month of January, the untouched Site 1 frost tube showed a frost line penetration to a depth of only 6 cm (Figure 7). The snow cover insulated the soil and severely retarded the penetration of the frost line. The insulating effect of snow was most noticeable with accumulations of fresh snow, as seen by the slight variations in the frost line (snow not removed).

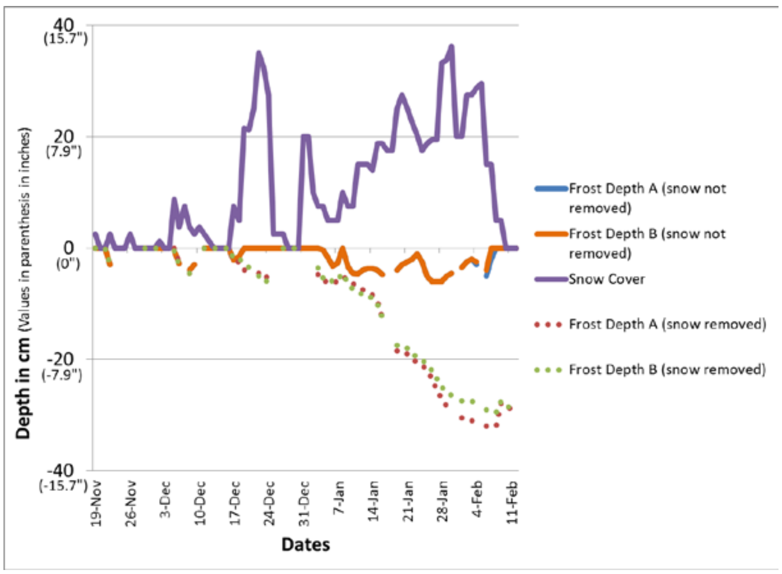

Figure 7. Snow cover and resulting frost depths (Site 1) while snowpack was present. Frost depths for areas where the snow cover has been routinely removed are shown for comparison.

Model development

Initial model

The model was constructed as a climate-only model. Soil type and soil derived parameters were not considered a model variable. The model assumed a grassy surface with a loam/soil and clay mixture and a saturated or nearly saturated soil. This is a safe assumption for Western New York and for much of the Midwest and Northeastern US. A Thornthwaite water budget, using a soil capacity of 15 cm equivalent precipitation/evaporation (typical of loam/clay soils), was run for the years 2008 to 2011 and results showed soil moisture was at capacity in the Buffalo area through the winter months. The model also assumes the absence of a snow cover.

To build the “rating curve,” the 2008–2010 accumulated degree-day plots (Figure 5) were sectioned at nick points where the rate (slope) of the accumulated degree-day changed. Each section’s rate, expressed as a FDD/day, was paired with the same time period on the freezing depth plot (Figure 6) and expressed as a freezing depth/day. Thus, a particular FDD/day rate corresponded to a matching freezing depth/day rate (expressed in cm). The paired rates were plotted on a rating curve and a best-fit line was applied (Figure 8). A polynomial best-fit equation was derived ($y = 0.0002[x^3] + 0.0003[x^2] + 0.0423[x] - 0.0143$), giving an R^2 value of 0.96. A number of curves were tested (including a linear curve), but the polynomial curve chosen offered both a good fit and made theoretical sense. Increased TDD/days did not lead to frost penetration, whereas increased FDD/day led to greater frost depths at a rate consistent with the observed data. The resulting best-fit equation was multiplied by the daily degree-days (either freezing degree-days or thawing degree-days), and the accumulated values were used to calculate the progression of the frost line over time. Modeled accumulated freezing depths that showed a positive value (soil not frozen) were truncated at zero.

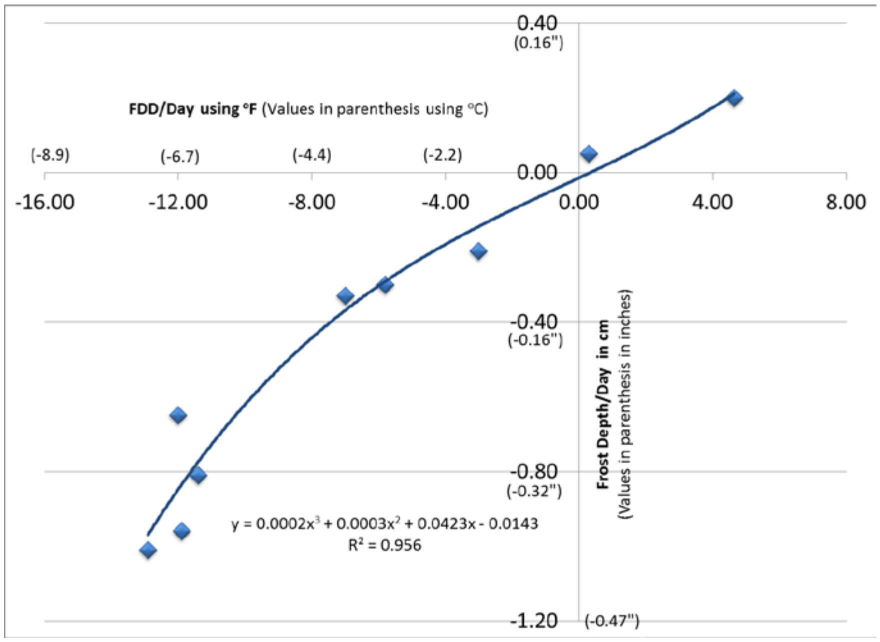

Figure 8. Modeling rating curve. FDD/day expressed as both Fahrenheit and Celsius FDDs.

An initial offset between measured and modeled values during the winter of 2009–2010 was due to a rapid penetration of the measured frost line down to 3.5 cm. It is believed that an extended period of below freezing minimum temperatures primed the first few centimeters of the surface soil to freeze to a depth greater than could be accounted for by the model. The first recorded FDD in 2009–2010 occurred in early December, about two and a half weeks later than the first FDD recorded for 2008–2009. An empirical adjustment was made to the model, where a soil freezing depth of 3.5 cm would be added to the initial modeled freezing depth only if the first FDD occurred as late as December.

The "initial model" outputs were plotted against accumulated freezing depths, as measured by the frost tubes (Figures 9 to 11). The "initial model" provided good agreement with the measured values for the first half of the winter season. During this period, absolute differences between modeled and measured values were usually within 5 cm. While the "initial model" behaved well over the first half of the winter season, in all cases it over-predicted frost depth in the latter half of the winter season. This misfit between modeled and measured values occurred at the cusp between frost line penetration and thawing at depth (Figures 9 to 11).

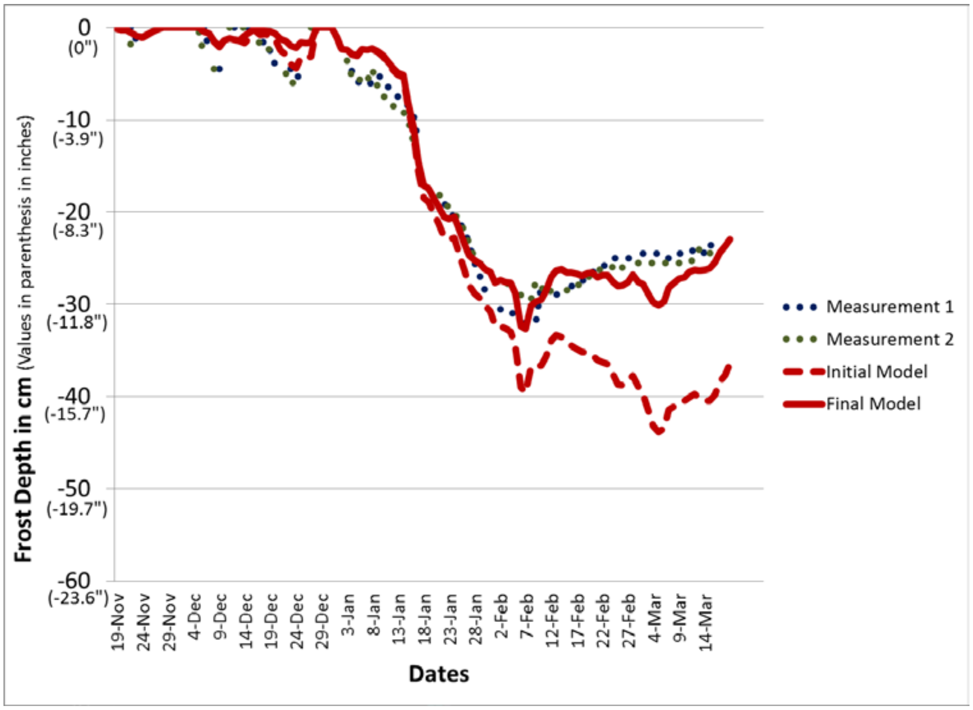

Figure 9. Site 1 measured and modeled soil frost progression and depth for 2008–2009.

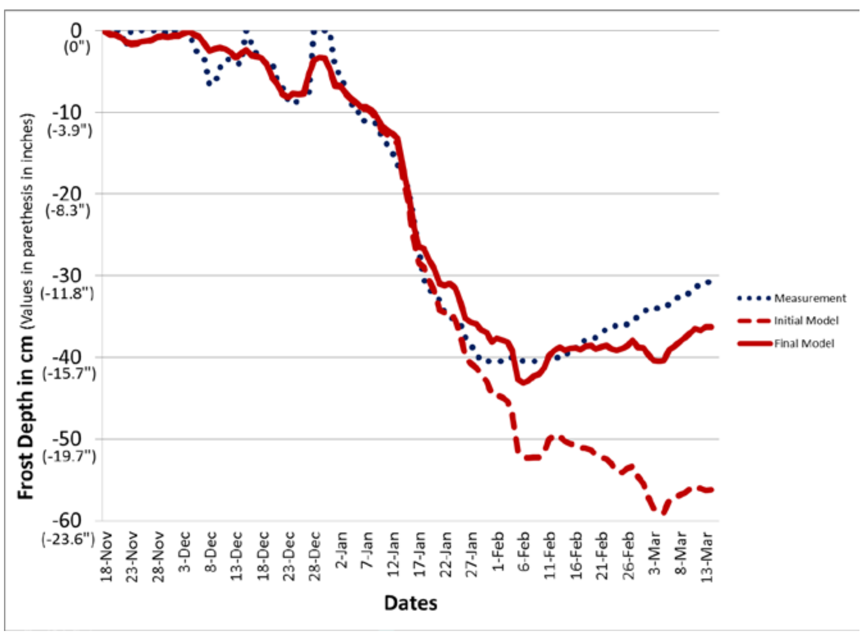

Figure 10. Site 2 measured and modeled soil frost progression and depth for 2008–2009.

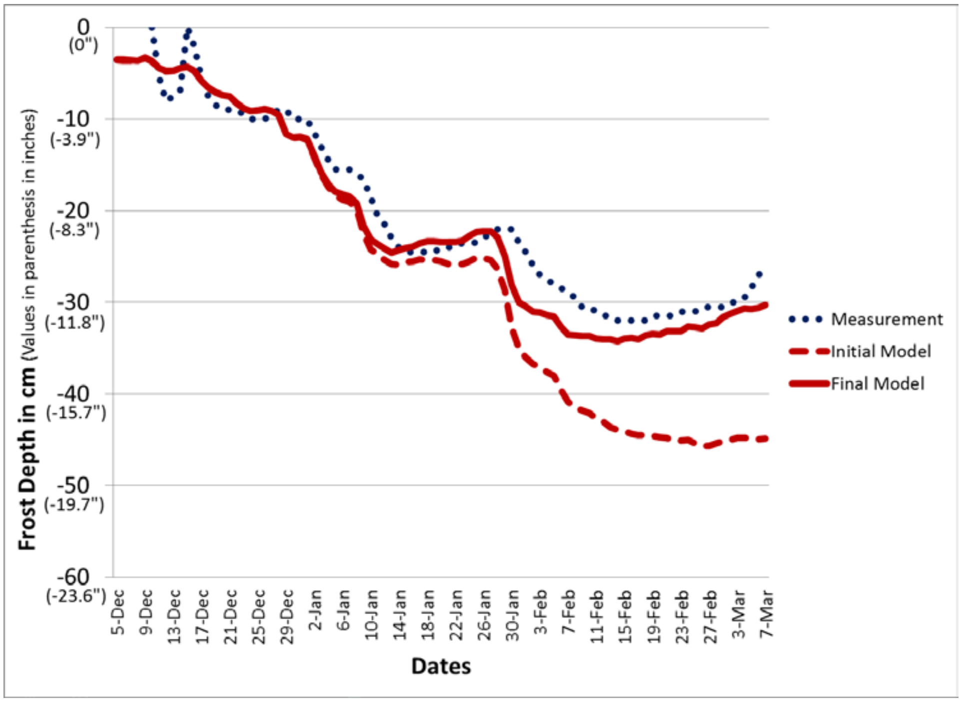

Figure 11. Site 2 measured and modeled soil frost progression and depth for 2009–2010.

Final model

The “initial model” assumed that air temperature was comparable to surface soil temperatures. This assumption may not be equally applicable on exposed ground in late February and March with lengthened days and an increasing solar angle. The freezing point of soil occurs where the soil heat loss exceeds heat gained from the atmosphere and from subsurface layers (geothermal heat flux). As air temperatures fall below freezing, increasing the FDD/day rate, the soil heat loss exceeds the upward heat flux from the subsurface; and the frost line penetrates deeper into the soil. However, in late winter/early spring, the longer periods of daylight and increased sun angle heat the exposed ground. The soil penetration of the FDD/day rate is lessoned, as it first must maintain freezing conditions at the surface. In other words, a greater FDD/day rate is required to maintain the frost line depth than that which would be required earlier in the season. As a result, the frost line penetration is slowed and later reversed due to the upward heat flux from subsurface layers. In keeping with our climate-only modeling approach, solar-related parameters can be used to determine the amount of heat transfer in place of a more complex soil-derived heat transfer equation.

To account for this change, a “day length” variable and a “sun angle” variable (referred to as the “solar adjustment”) was added to the “final model.” Day length and sun angle are dependent on seasonality and latitude and are easily obtained from the literature (United States Naval Observatory, 2010). The solar adjustment was calculated by dividing the day length of the shortest day of the year (Winter Solstice) by the day length for each day of the winter season. For example, at 43°N, the shortest day length is 9.0 hours while 15 February experiences 10.7 hours of daylight; thus the day length adjustment factor for 15 February is 0.84. A similar calculation was made for the daily sun angle. The daily solar adjustment was calculated by averaging the daily day length and daily sun angle adjustments. The daily output of the “initial model” was multiplied by the “solar adjustment” factor to calculate a modeled freezing depth (final model). Our approach, with sample data, is shown in a spreadsheet format (Table 2).

Table 2. Sample Model Calculations for 9 Days (FDD calculated in °F and depth in cm).

Initial Model							Final Model
Date	FDD	FDD * Rating Curve (calculated daily freezing depth)	Freezing Depth (FD)	Day-Length (DL)	Sun- Angle (SA)	Solar Adjustment (DL+SA)/2	FD * Solar Adj. (cm)
1-Feb	-14	-1.0965	-48.3916	0.9	0.79	0.845	-40.8909
2-Feb	-10	-0.6073	-48.9989	0.9	0.79	0.845	-41.4041
3-Feb	-18	-1.8449	-50.8438	0.9	0.78	0.840	-42.7088
4-Feb	-8	-0.4359	-51.2797	0.9	0.77	0.835	-42.8185
5-Feb	-7	-0.3643	-51.644	0.89	0.77	0.830	-42.8645
6-Feb	-4	-0.1915	-51.8355	0.89	0.76	0.825	-42.7643
7-Feb	-4	-0.1915	-52.027	0.89	0.75	0.820	-42.6621
8-Feb	-17	-1.6293	-53.6563	0.88	0.74	0.810	-43.4616
9-Feb	-16	-1.4335	-55.0898	0.88	0.73	0.805	-44.3473

For most of the winter, the modeled freezing depth differed little from that calculated by the “initial model,” as the adjustment factor was close to a value of 1.0. However, in the latter half of the winter season, the solar adjustment factor had greater weight, lessening the freezing depth predicted for a particular FDD/day rate. The outputs of the “final model” accurately reflected the cusp between frost line penetration and thawing at depth and accurately reflected deep soil thawing until the entire soil column was thawed (Figures 9 to 11). Agreement between measured and modeled outputs is also reflected in smaller temporal variations such as early to mid-January thaws.

Model validation

The “rating curve,” “initial freezing depth adjustment,” and a “solar adjustment” developed and tested for 2008–2010 were applied to the 2010–2011 winter season at Site 2 (Figure 12).

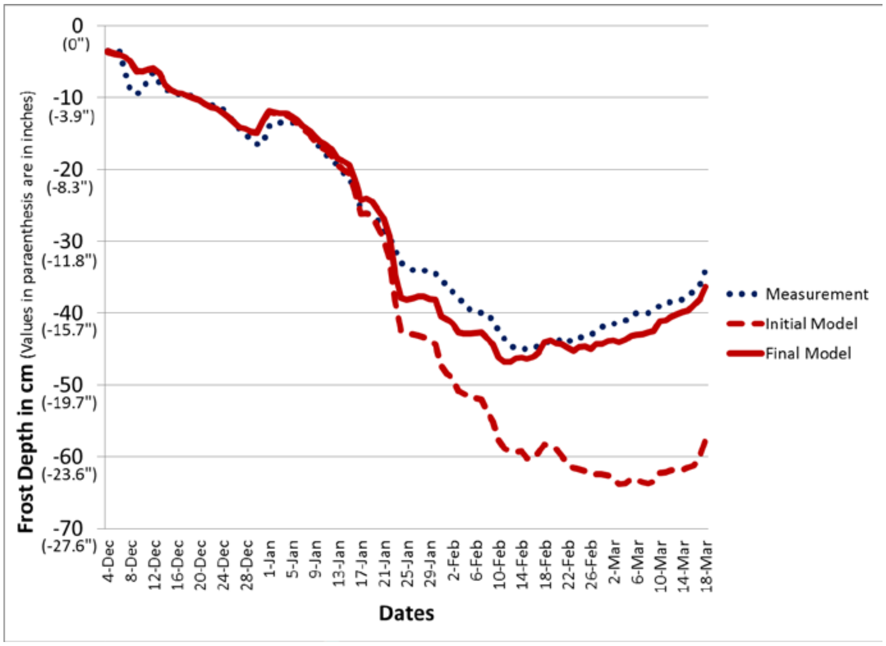

Figure 12. Site 2 measured and modeled soil frost progression and depth for 2010–2011.

The "final model" provided for an excellent fit between the modeled and measured values in 2010–2011. Overall, the modeled values correlate well with the measured values ($R^2 > 0.96$), with absolute differences of less than 5 cm. The 5 cm error (worst case) in this study is consistent with other soil-based model errors (DeGaetano et al., 2001). While the "final model" outputs account for the heating of soil by solar radiation, the "initial model," absent the solar adjustment factor, may be useful, reflect frost depth in perpetually shaded areas.

CONCLUSION

The CRREL-Gandahl frost tubes, as used in this study, worked well in their ease of use and in accurately predicting frost depth. Over three years of measurements (2008–2011), frost depths ranged from 29.5 to 45 cm, where snow cover was removed from the area around the frost tubes. For areas and times where the snow cover was untouched (Site 1 in 2008–2009), the frost penetration was limited to 6 cm, as snow cover insulated the soil and severely retarded the penetration of the frost line. A soil frost line model based on freezing degree rates (expressed as FDD/day), along with day length and sun angle adjustments, was developed assuming saturated or near saturated soils and the absence of a snow cover. The model parameters are easily obtained and the model output is easily calculated—amenable to a spread sheet format. The model output accurately predicted the maximum depth of frost line penetration, the cusp between frost line penetration and thawing at depth, and the rate of thawing at depth. Correlations between measured and modeled values were very strong ($R^2 > 0.96$) and absolute values were usually well within 5 cm. The frost line model is best used to predict frost depths in areas and times where snow cover is absent or when potential frost depth maximums are to be determined. In permanently shaded areas, the "initial model," absent the solar adjustment factors, may best reflect frost penetration depths.

The modeled approach offers a climate-only solution, independent of soil and freezing mechanics, to calculate real-time frost depth. While the model is empirically based and developed in a limited geographic area, it is anticipated that future research will further test the model and, perhaps more importantly, the concept behind the model. The climate-only approach has the potential to provide a quick and easily calculated real-time frost depth, or it can provide a quick screening of frost depths prior to the use of more complex soil-based models.

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